Characterization of Multi-Antenna GNSS, Multi-Sensor Attitude Determination for Stratospheric Balloon Platforms

Nathan Tehrani and Jason Gross

West Virginia University, Morgantown, WV, 26506, USA

In this paper, a multi-antenna Global Navigation Satellite System (GNSS), multi-sensor attitude estimation algorithm is outlined, and its sensitivity to various error sources is assessed. The attitude estimation algorithm first estimates attitude using multiple GNSS antennas, and then fuses a host of other attitude estimation sensors including tri-axial magnetometers, Sun sensors, and inertial sensors. This work is motivated by the attitude determination needs of the Antarctic Impulse Transient Antenna (ANITA) experiment, a high-altitude balloon-lofted science platform. In order to assess performance trade-offs of various algorithm configurations, the attitude estimation performance of various approaches is tested using a simulation that is based on recorded ANITA III flight data. For GNSS errors, attention is focused on multipath, receiver measurement noise, and carrier-phase breaks. For the remaining attitude sensors, different grades of sensor are assessed. Through a Monte-Carlo simulation, it is shown that, under typical conditions, sub-0.1 degree attitude accuracy is available when use multiple antenna GNSS data only, but that this accuracy can degrade to degree-level in some environments warranting the inclusion of additional attitude sensors to maintain the desired level of accuracy.

I. Introduction

This document outlines the development, simulation, and testing of an attitude determination algorithm. It is motivated by the requirements of the Antarctic Impulse Transient Antenna (ANITA) experiment. ANITA is an ongoing project that uses a balloon-lofted platform to detect radio impulses from high-energy particle collisions in the ice below. Ultra-high energy neutrinos (UHEN) and ultra-high energy cosmic rays (UHECR) have both been detected by IceCube, a ground-based neutrino observatory which uses detectors embedded in ice. ANITA, with its high operating altitude, can observe possible particle collisions in a significantly-larger volume of ice. The ANITA I, II, and III flight platforms have made successful radio transient discoveries. ANITA uses several feed-horn antennas with narrow observation beams and a high degree of pointing precision for each antenna. For any airborne sensing platform, the pointing accuracy is dependent on the accuracy of the onboard attitude solution. As such, a key to high pointing accuracy is a robust attitude-determination system.

Attitude determination using multi-antenna GNSS observations is an established process, first proposed by Cohen in 1991. It was also adapted for aircraft use and tested by the same author. Multi-antenna GNSS attitude determination has been tested on ground, waterborne, and flight vehicles, and the technology has matured to multiple commercially-available products. There has been considerable effort to simulate gyroscope-free attitude determination using 3-axis magnetometers, 2-axis Sun sensors, or both, for spacecraft applications. Highlights include the use of a magnetometer-only Sun-pointing algorithm by Ahn, 2003. This method did not include filtering and was used to estimate an attitude vector which was being corrected. Magnetometer-derived attitude was within 3° of gyroscope-derived truth for the entire investigated flight. Psiaki (1991) modeled an orbit- and attitude-determination algorithm. Using a 10nT 3-axis magnetometer

*Graduate Research Assistant, Department of Mechanical and Aerospace Engineering (MAE), Morgantown, WV, AIAA Student Member
†Assistant Professor, Department of MAE, Morgantown, WV, AIAA Senior Member

Copyright © 2017 by Nathan Tehrani, Jason Gross. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission.
and a 0.005° Sun sensor, this method showed less than 0.1° error in all axes. Crassidis (1996) created a Sun sensor and magnetometer Kalman filter and showed that a magnetometer-only attitude estimate is markedly improved (error reduced by approximately half) with the inclusion of Sun sensor data. The Balloon-borne Large Aperture Submillimeter Telescope for Polarimetry (BLASTPol) is a similar stratospheric platform that uses Kalman filtering of multi sensor data for post-flight attitude determination.

Multi-antenna GNSS has been used for remote sensing platforms since shortly after its proposal, and it is in use on multiple stratospheric balloon platforms. This paper outlines the design and performance evaluation of a GNSS-based attitude estimator that is then augmented with various other attitude sensors to offer a proposed algorithm for the ANITA project, or other similar balloon-based payloads.

This remainder of this paper is organized as follows. In Section II, the simulation environment and the sensor data simulation is discussed. In Section III, the design of the baseline estimation filter and attitude estimation filter are discussed. Section IV the performance of the GNSS-based and multi-sensor attitude estimators are presented and discussed. Section V summarizes this study’s findings and discusses future work.

II. Data Simulation

II.A. Flight Profile

The simulated flight data used in this study is based upon the recorded flight data of ANITA III. That is, to simulate a balloon flight, the onboard position and attitude solutions were accepted as truth for simulation purposes, and sensor readings with realistic measurement noise were simulated.

![Figure 1. Attitude profile used in this work.](image)

Figure 1 shows the Euler angle time histories during a two-hour segment of the ANITA III flight. As indicated in Fig. 1, the platform had a small (< 1°) oscillation in the roll and pitch axes and a constant rotation about the yaw axis.

II.B. GNSS Observables Simulation

For each simulation run, four GNSS receivers were simulated with baseline separations of one-meter each, such that they are arranged in a square configuration. That is, the antennas were placed according to the
following matrix $R_b$:

$$R_b = \begin{bmatrix} x_{2,b} & y_{2,b} & z_{2,b} \\ x_{3,b} & y_{3,b} & z_{3,b} \\ x_{4,b} & y_{4,b} & z_{4,b} \end{bmatrix}, \quad (1)$$

where $x_{i,b}, y_{i,b},$ and $z_{i,b}$ are the body-centric coordinates of the $i^{th}$ antenna $i = 1$ denoting the master antenna, as was done by Cohen in the first paper describing multi-antenna attitude determination.\(^5\) GNSS carrier-phase data was simulated for each flight profile at a rate of 10 Hz using the MATLAB SatNav Toolbox,\(^15\) which was modified by Watson et al. (2016)\(^16\) to include additional GNSS error sources.

A number of deterministic and non-deterministic error sources are associated with GNSS measurements.\(^17\) Fortunately, for attitude estimation applications, several of the primary GNSS error sources, including satellite and receiver clock biases and atmospheric delays, are canceled through the use of double differenced GNSS observations.\(^17\) However, two important error sources, namely multipath reflections and carrier-phase breaks (AKA cycle-slips) remain present. In particular, when a metallic object reflects a GNSS signal onto the antenna, the multiple paths induce errors.\(^17\) This could be a large problem on balloon-based scientific platforms, as the antennas are spaced closely and in close proximity to science payload. Thermal measurement noise in the receiver is another error source; it is actually amplified by double differencing GNSS data. As such, for this simulation study, multipath, carrier-phase breaks, and receiver thermal errors were assessed with respect to their effect on the attitude estimator’s performance using the distributions indicated in Table 1.

II.C. Inertial Measurement Simulation

In addition to GNSS measurements, inertial measurement unit data was simulated for each flight profile and data at a sampling rate of 200 Hz. In particular, four grades of IMU tri-axial rate gyroscope and accelerometers were simulated assessed. In this case, ideal gyroscope readings were generated by accepting the truth attitude solution of the ANITA III flight. These ideal measurements were then polluted with both a time-varying bias and a white noise component. The magnitude of these two noise terms were selected based on the grade of the inertial sensors assumed, which were varied as indicated in Table 1.

II.D. Sun Sensor & Magnetometer Simulation

Two-axis Sun-sensor data and tri-axial magnetometer data were also simulated for each flight based on the measurement models and uncertainties of the sensors current installed on the ANITA IV balloon. In particular, the apparent Sun position and the Earth’s magnetic field along the flight profile were calculated and sensor measurements were simulated by polluting these true values with random noise based on the measurement noises quoted by the manufacturers’ spec. sheets as indicated in Table 1.

The magnetometer data consists of magnetic field intensity measurements ($B_b$) in three orthogonal directions corresponding to the North, $N$, East, $E$, and down $D$ axes in the body frame, $b$. This begins with $B_E$, a vector constraining the simulated magnetic field intensities in the navigation frame, generated at each location along the flight path:

$$\vec{B}_E = \begin{pmatrix} B_{b,N} \\ B_{b,E} \\ B_{b,D} \end{pmatrix}. \quad (2)$$

Body-frame magnetic field measurements are generated by multiplying truth attitude (represented by the direction-cosine matrix $C_{b}^n$) by the navigation-frame magnetic field:

$$\vec{B}_b = C_{b}^n \vec{B}_E. \quad (3)$$

With three contributing error sources added: hard and soft iron errors and measurement noise, in a simplified method as described by Gebre-Egziabher et. al.:\(^18\)

$$\hat{B} = A_{si} \bar{B}_b + \bar{B}_{hi}. \quad (4)$$
where $A_{si}$ is a $3 \times 3$ matrix which describes the soft-iron error effect and $\vec{B}_{hi}$ is a $3 \times 1$ vector containing the hard-iron offset, a magnetic field generated by ferromagnetic material on the platform. For this study, nominal values for $A_{si}$ and $\vec{B}_{hi}$ were used, based on the calibrations in the Gebre-Egziabher paper. Simulated measurement noise was then added to $\vec{B}$, corresponding to precision level of the modeled magnetometer.

The simulated Sun sensor data consists of solar incidence angles $\zeta_X$ and $\zeta_Y$ relative to the two horizontal body-frame axes $X_b$ and $Y_b$. These were generated using the apparent solar azimuth $\theta_{Sun}$ and elevation $\phi_{Sun}$ calculated for each epoch of the flight duration. First, the solar azimuth and elevation values are transformed into a unit vector representing the Sun’s position in the sky with respect to the navigation frame, $n$:

$$V_{Sun,n} = \begin{pmatrix} Sun_{x,n} \\ Sun_{y,n} \\ Sun_{z,n} \end{pmatrix}. \quad (5)$$

This unit-vector is then transformed using the nav-to-body direction cosine matrix, $C^b_n$:

$$V_{Sun,b} = C^b_n V_{Sun,n} \quad (6)$$

and the solar incidence angles $\zeta_X$ and $\zeta_Y$ are then calculated:

$$\zeta_X = \pi/2 + \text{atan2}(Sun_{z,b}/Sun_{x,b}), \quad (7)$$

$$\zeta_Y = \pi/2 + \text{atan2}(Sun_{z,b}/Sun_{y,b}); \quad (8)$$

where \text{atan2} is the four-quadrant tangent inverse.

As with the magnetometer measurements, simulated measurement noise was added to the Sun sensor measurements. However, in the case of a Sun sensor, as measurement noise increases at low solar elevations, the measurement noise was scaled according to solar elevation angle. Sun sensor measurements were simulated at 10Hz intervals.

II.E. Simulation Overview

For this study, a total of 50 one-hour flight profiles were simulated in a Monte-Carlo manner. In particular, the ECEF starting positions, magnitude of GNSS error sources, and quality of IMU, Magnetometer and Sun sensor data were varied as indicated in Table 1. Note that by randomly varying the starting location, the GNSS constellation satellite geometry was randomized as well.

III. Attitude Estimation

III.A. Algorithm Overview

Figure 2 shows the overall algorithm used. First, a carrier-phase differential GNSS filter, as detailed in Section III.B, estimates the baselines between antennas. Next, this information is used as a measurement update for a GNSS-only multiple antenna attitude estimator as described in Section III.C, in which the attitude estimates are smoothed by assuming typical low-dynamic balloon flights. Finally, the resulting estimated attitude state is optionally fused with a multi-sensor estimator that also incorporates inertial, magnetometer, and Sun sensor data, as discussed in Section III.D.
Table 1. Sensor Error-Source Monte-Carlo Simulation Distribution Parameters

<table>
<thead>
<tr>
<th>Error-Sources</th>
<th>Model Parameters</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Noise</td>
<td>( \sigma_p = 0.32 m ), ( \sigma_\phi = 0.16 \lambda )</td>
<td>linear scale factor randomly selected between [0,1]</td>
</tr>
<tr>
<td>Multipath</td>
<td>1.0 intensity: ( \sigma = 0.4 m ), ( \tau = 15 \text{sec} )</td>
<td>linear scale factor randomly selected between [0,2]</td>
</tr>
<tr>
<td>Tropospheric Delay</td>
<td>Percent of error assumed handled by broadcast correction</td>
<td>Modified Hopfield with linear scale factor randomly selected between [0.95,1.05]</td>
</tr>
<tr>
<td>Ionospheric Delay</td>
<td>First order ionospheric effects mitigated with dual-frequency</td>
<td>linear scale factor randomly selected between [0.7,1]</td>
</tr>
<tr>
<td>Carrier phase break</td>
<td>Likelihood set to 1 phase break per 24 minute to 1 phase break per 240 minutes</td>
<td></td>
</tr>
<tr>
<td>Gyroscope</td>
<td>In-run Bias ( \sigma = 9.6 e^{-6 \text{ rad sec}^{-1}} ), ( ARW = 0.2 \text{ deg \sqrt{hr}} )</td>
<td>Scaled Honeywell HG1700AG72 SF = ((\frac{1}{50}, \frac{1}{200}, \frac{1}{400}))</td>
</tr>
<tr>
<td>Sun Sensor</td>
<td>Zenith measurement noise ( \sigma = 0.1 \text{ deg} )</td>
<td>Scaled SolarMEMS ISSDX-60 SF = ((1, 2, 3, 4))</td>
</tr>
<tr>
<td>Magnetometer</td>
<td>Measurement noise ( \sigma = 2.67 \text{ nT} ) ( A_{si} ) terms scaled between ([0.005, 0.01]) ( B_{hi} ) terms scaled between ([25\text{nT}, 50\text{nT}])</td>
<td>Scaled ST LSM9DS0 SF = ((1, 5, 10))</td>
</tr>
</tbody>
</table>

Figure 2. Block diagram showing the three main estimators: baseline-estimation filter, GNSS-only attitude estimator, and multi-sensor attitude estimator.

III.B. Antenna Baseline Estimation Filter

A Kalman filter is used to estimate the relative position between each of the antennas and a single master antenna at each of the 10 Hz measurement epochs. In particular, this Kalman filter uses Carrier-phase Differential GNSS (CD-GNSS) measurements to estimate the relative position vectors between the antennas.\(^{17}\) The state vector, \( x \), for this filter consists of the relative position vector components between antenna’s A
and $B$, and a set of double-difference carrier-phase biases $N_{A,B}$.

$$
\begin{bmatrix}
x_{A,B} \\
y_{A,B} \\
z_{A,B} \\
N_{A,B}^{1,k} \\
\vdots \\
N_{A,B}^{j,k}
\end{bmatrix}
$$

(9)

The measurement models used to model the double-differenced carrier-phase observables follow the same approach outlined in,\textsuperscript{19} as is discussed next.

First, the model for an undifferenced GNSS carrier-phase measurement, $\phi$, (with units of carrier cycles) is given as:\textsuperscript{17}

$$
\phi = \frac{1}{\lambda} [r + I_\phi + T_\phi] + \frac{c}{\lambda} (\delta t_u - \delta t_s) + N + \epsilon_\phi,
$$

(10)

where $\lambda$ is the wavelength corresponding to the frequencies $L1$ and $L2$ and expressed in meters. The geometric range $r$ between the receiver and GNSS satellite is also expressed in meters, as are the ionospheric and tropospheric delays $I$ and $T$. The speed of light $c$ is expressed in meters per second. The clock biases of the receiver and satellite, $\delta t_u$ and $\delta t_s$, respectively, are expressed in seconds. The un-modeled error sources, which include multipath and thermal noise, are included in $\epsilon$ in units of meters.

First, carrier-phase measurements for the master antenna $A$ (antenna 1) and $B$ (antennas 2, 3, or 4) are differenced to form single-differenced phase measurements:

$$
\Delta \phi_{jA,B} = \frac{1}{\lambda} r_{jA,B} + \frac{c}{\lambda} (\delta t_u - \delta t_s) + N_{jA,B} + \epsilon_{jA,B}.
$$

(11)

Within Eq. 11, due to the very short baseline separation between the antennas, the atmospheric delays completely cancel along with the any satellite clock bias and ephemeris errors. Next, the single differenced measurements are then differenced between satellites. For example, between satellite $j$ and a reference satellite $k$:

$$
\nabla \Delta \phi_{jA,B} = -\frac{1}{\lambda} (1_j - 1_k) \mathbf{T}_{jA/B_k} + N_{jA,B} + \epsilon_{jA,B}.
$$

(12)

where the remaining receiver clock bias errors are eliminated, leaving only the unknown phase bias $N_{jA,B}$, which is known to be an integer.

Within this filter, the measurement vector, $z$, consists of double-differenced phase measurements for each satellite relative to the reference satellite, including measurements for each the $L1$ and $L2$ frequencies:

$$
z = \begin{bmatrix}
\nabla \Delta \phi_{L1A,B}^{i\ldots n,k} \\
\nabla \Delta \phi_{L2A,B}^{i\ldots n,k}
\end{bmatrix}.
$$

(13)

In parallel with this Kalman filter, the floating point estimated phase bases, $N_{jA,B}$ and their estimated error-covariance are fed into and integer ambiguity resolution algorithm. In particular, the Least-squares AMBignuity Decorrelation Adjustment (LAMBDA) method\textsuperscript{20} is used to determine the integer biases and adjust the estimated relative positions.

**III.C. GNSS-only Attitude Determination**

Once the antenna relative baselines with respect to a master antenna are estimated using the baseline estimation filter, an ECEF antenna relative position matrix, $R_{ECEF}$ is generated at each epoch by vertically concatenating the estimate relative vectors of each of non-master antenna, as adopted from Cohen:\textsuperscript{5}

$$
R_{ECEF} = \begin{bmatrix}
x_{2,ECEF} & y_{2,ECEF} & z_{2,ECEF} \\
x_{3,ECEF} & y_{3,ECEF} & z_{3,ECEF} \\
x_{4,ECEF} & y_{4,ECEF} & z_{4,ECEF}
\end{bmatrix}
$$

(14)
This matrix is then fed to a parallel estimator to estimate the platform attitude given the antenna baseline vectors, in which the state vector $x$ contains the attitude state expressed in Euler angles the represent the rotation from the body to navigation-frame:

$$x = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix}. \quad (15)$$

Using Horn's Method, the rotation matrix between body and Earth-fixed frames is found using the estimated the ECEF configuration, $R_{\text{ECEF}}$, and the known body-axis antenna configuration $R_b$. Horn’s method is a variant of the sum of least squares, where $R_b$ and $R_{\text{ECEF}}$ are both centered about their respective centroids, yielding $R'_b$ and $R'_{\text{ECEF}}$, where a matrix, $M$, is equal to the product of the two centered coordinate matrices:

$$M = R'_b \ast R'_{\text{ECEF}}. \quad (16)$$

The elements of this matrix are defined as follows:

$$N = \begin{bmatrix} (S_{xx} + S_{yy} + S_{zz}) & S_{yz} - S_{zy} & S_{zx} - S_{xz} & S_{xy} - S_{yx} \\ S_{yz} - S_{zy} & (S_{xx} - S_{yy} - S_{zz}) & S_{zy} + S_{yx} & S_{xx} + S_{zz} \\ S_{zx} - S_{xz} & S_{zy} + S_{yx} & (S_{xx} + S_{yy} - S_{zz}) & S_{yx} + S_{zy} \\ S_{xy} - S_{yx} & S_{zy} + S_{yx} & S_{yz} + S_{zy} & (S_{xx} - S_{yy} + S_{zz}) \end{bmatrix}. \quad (17)$$

The eigenvector $V$ corresponding to the highest eigenvalue of $N$ is normalized to form a unit quaternion, and the imaginary component is omitted. The resulting real quaternion is converted to a rotational matrix, $C'_b$, the body-frame to Earth-frame transformation matrix, which is in turn multiplied by the Earth-frame to navigation-frame transformation $C'_E$. This is converted into the measured Euler angles, and the filter state is updated.

This filter’s measurement update consists of the Euler angles from the baseline vectors. The error-state covariance matrix $P$ is initialized as a diagonal matrix containing the error magnitudes for each Euler angle, in this case $0.1^\circ$. The process noise matrix $Q$ is set as the identity matrix, as roll, pitch, and yaw rates are very low for this platform.

### III.D. Multi-Sensor Attitude Unscented Kalman Filter

Finally, a third Kalman filter estimator is used for attitude determination using all sensor data. In this step, an unscented Kalman filter (UKF) was chosen for its ability to handle the nonlinear transformation between platform attitude and solar incidence angles in the Sun sensor measurements. The details of the UKF implementation followed in this study are offered the tutorial paper by Rhudy and Gu and as such, these details are not discussed in detail herein. In this paper, an outline of the state vector, state prediction $f(x)$, and observation functions $h(x)$ for each measurement update are discussed.

The state vector, $x$ estimated in the Multi-Sensor filter is given as:

$$x = \begin{pmatrix} \phi \\ \theta \\ \psi \\ b_p \\ b_q \\ b_r \end{pmatrix}. \quad (19)$$

where $\phi$, $\theta$, and $\psi$ are the platform’s roll, pitch and yaw, and $b_{p,q,r}$ are the time-varying biases of the IMU’s roll rate, $p$, pitch rate, $q$, and yaw rate $r$ gyroscopes.
Within the UKF framework, at each epoch, the state vector is expanded into a group of \(2L+1\) sigma points, \(\chi\), where \(L = 6\) is the length of the estimated state vector. For each group of sigma points \(l\), the attitude states are predicted by integrating the IMU gyro data through the attitude kinematic equations:

\[
f(\phi, \theta, \psi) : \begin{bmatrix} \phi_i \\ \theta_i \\ \psi_i \\ \phi_{i-1} \\ \theta_{i-1} \\ \psi_{i-1} \end{bmatrix} = \begin{bmatrix} \phi_{i-1} \\ \theta_{i-1} \\ \psi_{i-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} t(\theta_{i-1}) s(\phi_{i-1}) & t(\theta_{i-1}) c(\phi_{i-1}) & -s(\phi_{i-1}) c(\theta_{i-1}) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \Delta t,
\]

\[(20)\]

where \(s(\cdot)\) represents sine, \(c(\cdot)\) represents cosine, and \(t(\cdot)\) represents tangent. Furthermore, \(\phi_{i-1}, \theta_{i-1}, \) and \(\psi_{i-1}\) are the previous epoch’s roll, pitch, and yaw sigma points, which are the first three elements of each column of \(\chi\), and \(b_{p,q,r}\) are the sigma points corresponding to the IMU bias states, which are predicted as random walk parameters.

\[
f(b_{p,q,r}) : \begin{bmatrix} b_{p,i} \\ b_{q,i} \\ b_{r,i} \\ b_{p,i-1} \\ b_{q,i-1} \\ b_{r,i-1} \end{bmatrix} = \begin{bmatrix} b_{p,i-1} \\ b_{q,i-1} \\ b_{r,i-1} \\ w_{bp} \\ w_{bq} \\ w_{br} \end{bmatrix}
\]

\[(21)\]

The measurement-prediction matrix \(\Psi\) is populated by the predicted measurement vectors using each set of sigma-points in \(\chi\). Because measurements occur at different rates in this filter, it is necessary to have different measurement updates occur at different rates. For epochs coinciding with Sun sensor and GNSS attitude measurements, each column \(\Psi_i\) is as follows:

\[
\Psi_i = \begin{bmatrix} B_{b,x} \\ B_{b,y} \\ B_{b,z} \\ \phi' \\ \theta' \\ \psi' \end{bmatrix}.
\]

\[(22)\]

where \(B_b, \phi_X,\) and \(\phi_Y\) are predicted magnetometer and Sun sensor measurements based on the \(i^{th}\) sigma point. The observation models, \(h(x)\) used to predict the magnetometer and Sen sensor measurements based upon estimate attitude sigma points are identical to those used to generate the data as discussed in Section II, with the exception that no magnetometer biases are estimated in the filter. That is, the observation equations use \(\hat{C}_n^b\), the direction-cosine representation of the predicted attitude states \(\phi, \theta,\) and \(\psi\):

\[
h_B(\phi, \theta, \psi) : \hat{B}_b = \hat{C}_n^b B_n.
\]

\[(23)\]

\[
V_{Sun,b} = \hat{C}_n^b V_{Sun,n}
\]

\[(24)\]

\[
h_{\phi_X}(\phi, \theta, \psi) : \phi_X = \pi/2 + atan2(Sun_{z,b}/Sun_{x,b});
\]

\[(25)\]

\[
h_{\phi_Y}(\phi, \theta, \psi) : \phi_Y = \pi/2 + atan2(Sun_{z,b}/Sun_{y,b});
\]

\[(26)\]

As GNSS attitude and Sun sensor measurements occur at at 10Hz rate, the remaining (50Hz) measurement updates consist only of magnetometer measurement predictions:

\[
\Psi_i = \begin{bmatrix} B_{b,x} \\ B_{b,y} \\ B_{b,z} \end{bmatrix},
\]

\[(27)\]
The measurement update matrix $z$ consists of the simulated sensor measurement at each filter epoch. These are similar in form to the columns of $\Psi$:

$$
\begin{bmatrix}
B_{b,x} \\
B_{b,y} \\
B_{b,z} \\
\phi_{GNSS} \\
\theta_{GNSS} \\
\psi_{GNSS}
\end{bmatrix},
$$

(28)

for filter epochs with GNSS, magnetometer, and Sun sensor measurements, and

$$
\begin{bmatrix}
B_{b,x} \\
B_{b,y} \\
B_{b,z}
\end{bmatrix},
$$

(29)

for epochs with magnetometer measurements only.

### III.E. Assumed Stochastic Parameters

The Kalman filter process noise, $Q$ and measurement noise, $R$, and initial error-covariance $P_0$, assumptions selected for the differential GNSS baseline estimator are outlined in Table 2:

<table>
<thead>
<tr>
<th>Filter Parameter</th>
<th>Assumed Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State error covariance $P_0$</strong></td>
<td>Baseline states: 1 m</td>
</tr>
<tr>
<td></td>
<td>Ambiguity states: 225 m</td>
</tr>
<tr>
<td><strong>Measurement noise covariance $R$</strong></td>
<td>$\sigma_\phi=4\cdot10^{-4}$ m</td>
</tr>
<tr>
<td><strong>Process noise covariance $Q$</strong></td>
<td>Attitude states: In-run Bias $10^{-2} \text{ m}/\sqrt{s}$</td>
</tr>
<tr>
<td></td>
<td>Ambiguity states: 0 m$/\sqrt{s}$</td>
</tr>
</tbody>
</table>

The multi-sensor, unscented Kalman filter was developed to run in multiple modes (GNSS-inertial, inertial, magnetometer and Sun sensor only, and all sensors). The different modes required different tuning parameters for adequate performance. These are outlined in Table 3:

<table>
<thead>
<tr>
<th>Filter Parameter</th>
<th>Assumed Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State error covariance $P_0$</strong></td>
<td>INS+Mag+SS: $10^{-2}$ deg. attitude states, $10^{-6}$ deg. bias states;</td>
</tr>
<tr>
<td></td>
<td>INS+GNSS: $10^{-3}$ deg. attitude states, $10^{-5}$ deg. bias states;</td>
</tr>
<tr>
<td></td>
<td>INS+all: $10^{-2}$ deg. attitude states, $10^{-6}$ deg. bias states</td>
</tr>
<tr>
<td><strong>Measurement noise covariance $R$</strong></td>
<td>$\sigma_{Mag}=25\text{nT}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{Sun}=0.1$ deg</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{GNSS}(\phi,\theta)=0.1$ deg $\cdot ADOP$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{GNSS}(\psi)=0.01$ deg $\cdot ADOP$</td>
</tr>
<tr>
<td><strong>Process noise covariance $Q$</strong></td>
<td>Attitude states: In-run Bias $10^{-2} \frac{\text{rad}}{\sqrt{s}}$</td>
</tr>
<tr>
<td></td>
<td>Bias states: ARW $10^{2} \frac{\text{rad}}{\sqrt{s}}$</td>
</tr>
</tbody>
</table>
IV. Results

IV.A. Results Overview

To summarize the results of the Monte-Carlo study, Figure 3 shows the cumulative distribution of the 3D attitude error $= \sqrt{\phi^2 + \theta^2 + \psi^2}$ for the various filter configurations over the 50 simulated flights. As is evident in Figure 3, in general, the additional sensors yield improved performance. Furthermore, there is a clear advantage to adding GLONASS data into the estimator. When all sensors are fused, attitude estimation less than 0.2 degrees is available for approximately 75% of the data. The CDF plot cuts off at two degrees as a small number of trials for which the filter diverged and are not shown for clarity.

IV.B. GNSS-only Attitude Filter

The GNSS-only attitude filter was run in two modes, the first using GPS data only, and the second adding GLONASS observables. The pitch, roll, and heading error statistics for both filter modes are presented in Tables 4 and 5. These results include two simulations for which the baseline filter solution failed to converge, presumably due to carrier-phase break.

Table 4. GPS-only Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>Roll (deg.)</th>
<th>Pitch (deg.)</th>
<th>Heading (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.0874</td>
<td>0.0867</td>
<td>0.0165</td>
</tr>
<tr>
<td>Max</td>
<td>106.0718</td>
<td>40.4702</td>
<td>221.8536</td>
</tr>
<tr>
<td>Mean</td>
<td>4.7867</td>
<td>1.5972</td>
<td>6.0898</td>
</tr>
<tr>
<td>Median</td>
<td>0.4281</td>
<td>0.4212</td>
<td>0.1611</td>
</tr>
</tbody>
</table>
Table 5. GPS+GLONASS Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>Roll (deg.)</th>
<th>Pitch (deg.)</th>
<th>Heading (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.0405</td>
<td>0.0500</td>
<td>0.0112</td>
</tr>
<tr>
<td>Max</td>
<td>103.6268</td>
<td>51.9286</td>
<td>347.5524</td>
</tr>
<tr>
<td>Mean</td>
<td>4.8948</td>
<td>2.7352</td>
<td>11.5969</td>
</tr>
<tr>
<td>Median</td>
<td>0.2833</td>
<td>0.2661</td>
<td>0.0958</td>
</tr>
</tbody>
</table>

Using GLONASS as well as GPS satellites yielded a median performance improvement of 40 percent lower attitude error. Considering ANITA’s Antarctic flight regime, fewer GNSS satellites are observable, and these are seen at lower elevations. This can negatively impact the Geometric Dilution of Precision (GDOP), a metric that describes the geometric diversity of satellite-receiver vectors. The attitude dilution of precision, as proposed by Yoon (2001) is a similar metric which assesses the ability to measure Euler angles. It is defined as:

\[
ADOP = \sqrt{\text{tr}[(nI - SS^T)^{-1}]},
\]

where \(n\) is the number of satellites in view, \(I\) is the \(3 \times 3\) identity matrix, and \(S\) is a \(3 \times N\) matrix comprising the unit vectors to each satellite, including the reference satellite. A variable starting location was used to investigate the effect of the lower GDOP and ADOP at high latitudes. Figure 4 shows error performance using GPS satellites only and using both GPS and GLONASS satellites, as well as the ADOP calculated in each case, for a polar flight profile:

Figure 4. Comparison between GPS-only mode and GPS+GLONASS mode for a polar flight profile.

IV.C. GNSS Multi-sensor Attitude Filter

Tables 6, 7, and 8 present overall error statistics for the 50 trials (including the two convergence failures) for the GNSS+INS, GNSS+ All sensors, and All sensors without GNSS, respectively. The filter failed to converge two times when the magnetic hard-iron bias was close to the magnetic process noise parameter in the measurement covariance matrix.
Table 6. GNSS+INS Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>Roll (deg.)</th>
<th>Pitch (deg.)</th>
<th>Heading (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.0332</td>
<td>0.0339</td>
<td>0.0160</td>
</tr>
<tr>
<td>Max</td>
<td>104.1555</td>
<td>51.9770</td>
<td>347.5273</td>
</tr>
<tr>
<td>Median</td>
<td>0.1078</td>
<td>0.1064</td>
<td>0.0505</td>
</tr>
</tbody>
</table>

Table 7. GNSS+INS+Mag+SS Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>Roll (deg.)</th>
<th>Pitch (deg.)</th>
<th>Heading (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.0277</td>
<td>0.0220</td>
<td>0.0209</td>
</tr>
<tr>
<td>Max</td>
<td>6.14e+4</td>
<td>0.0189e+4</td>
<td>3.57e+4</td>
</tr>
<tr>
<td>Median</td>
<td>0.1078</td>
<td>0.1064</td>
<td>0.0505</td>
</tr>
</tbody>
</table>

Table 8. INS+Mag+SS Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>Roll (deg.)</th>
<th>Pitch (deg.)</th>
<th>Heading (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.0277</td>
<td>0.0220</td>
<td>0.0210</td>
</tr>
<tr>
<td>Max</td>
<td>9.79e+5</td>
<td>0.0012e+5</td>
<td>0.0032e+5</td>
</tr>
<tr>
<td>Median</td>
<td>0.0779</td>
<td>0.0723</td>
<td>0.0736</td>
</tr>
</tbody>
</table>

In these tables, it is clear that that using additional sensors in addition to GNSS can markedly improve performance. For example, Figure 5 shows the attitude estimation error for one example trial, in which the GNSS-only attitude is shown alongside the multi-sensor filters for comparison.

![Figure 5. Roll, pitch, and heading errors for multi-sensor filter in GNSS+INS mode, GNSS+INS+Mag+SS mode, with GNSS-only filter result for comparison.](image)

Of great interest is the algorithm’s ability to handle carrier-phase breaks. For example, phase breaks could occur due to radio-frequency interference, such as during a data transmission over the Iridium satellite...
constellation which operates very close to the GPS L1 frequency.\textsuperscript{26} When a carrier-phase break occurs, it can fortunately be detected easily by a data editor.\textsuperscript{27} As such, whenever this occurs, the baseline estimation filter re-sets the error-covariance for the impacted carrier-phase ambiguities to a large value. The result is a momentary spike in attitude error, not longer than five filter time steps, but often with multi-degree magnitude. The multi-sensor filter attitude determination performance was lower across the range of phase break likelihoods as shown in Figure 6. Notably, the multi-sensor UKF yielded a low error-level attitude solution for the two trials with GNSS-attitude convergence failure.

Also of interest is the filter’s performance with high receiver measurement thermal noise and multipath errors. Figures 7 and 8 show that the multi-sensor filter yields lower-magnitude errors than the GNSS-only filter across both error scale ranges. Although an increasing level of multipath error did not noticeably affect the result of the GNSS-only filter performance, the multi-sensor filter performed better in nearly all trials.

The GNSS-only filter had a much greater sensitivity to the thermal error noise factor, and the multi-sensor filter had a more-consistent error level for the thermal error range. These plots do not include trials which failed to converge.

Figure 6. RMS attitude vs. phase break likelihood for each trial.
Sensitivity to the ionospheric and tropospheric error contribution to the GNSS errors was not considered, as the short baseline between antennas led to cancellation of those error sources.

V. Conclusion and Future Work

This study outlined the design and testing of a GNSS-based attitude determination algorithm, as well as its augmentation with additional sensor data. GNSS-only attitude solutions are consistently improved when GLONASS satellites are included in addition to GPS, owing to more observables and lower dilution.
of precision (especially in polar regions). Furthermore, adding inertial measurements, Sun sensor and magnetometer data further improves attitude-determination performance and reliability. This simulation study will be followed up with set of UAV flight test experiment. Three GNSS receivers and antennas have been mounted on a UAV platform as shown in Figure 9, along with a magnetometer, Sun sensor, and an IMU.

![Figure 9. WVU Phastball Zero UAV outfitted with GNSS receivers.](image)

This will allow for test and evaluation of the proposed algorithm using actual flight data, which will include a high grade GNSS/INS as a reference solution.

Acknowledgments

This work was supported in part by NASA West Virginia Space Grant Consortium and the National Geospatial Intelligence Agency Academic Research Program grant # HM0476-15-1-0004. Approved for NGA public release under case number 17-053.

References