UGV-to-UAV Cooperative Ranging for Robust Navigation in GNSS-Challenged Environments

Victor O. Sivaneri, Jason N. Gross

Abstract

This paper considers cooperative navigation between an Unmanned Aerial Vehicle (UAV) operating in a GNSS-challenged environment with an Unmanned Ground Vehicle (UGV), and focuses on the design of the optimal motion of the UGV to best aide the UAV’s navigation solution. Our approach reduces the uncertainty of a UAV’s navigation solution through the use of peer-to-peer radio ranging from a cooperative UGV, whose location is designed to improve positioning geometry for the UAV. Two novel cooperative strategies and two different estimation strategies for the UGV to assist a UAV are developed and compared. Through the use of a realistic simulation environment, it is shown that employing UGV-to-UAV cooperative navigation can reduce the positioning error of a UAV that is operating in a GNSS-challenged environment, from approximately 1–meter–level to approximately 10–cm–level 3D positioning error.

I. INTRODUCTION

Collaborative or cooperative navigation research has been increasingly active in recent years, especially in support of military operations and Intelligent Transportation Systems (ITS) [1]. This is due to the fact that these applications are oftentimes confronted with GNSS-challenged environments. For example, in military situations, vehicles may go from situations where GNSS is readily available to completely denied (e.g., being jammed). In the context of ITS, urban canyons often lead to extreme multipath, complete GNSS blockages and/or the reception of non-Line of Sight (LOS) signals, which can lead to position errors as large as hundreds of meters [2]. These scenarios are a few of the better-known cases in which collaborative or cooperative navigation would be beneficial. Other potential applications include the use of small quadrotor unmanned aerial vehicles (UAVs) for bridge inspection or structural health monitoring [3], UAVs for surveillance applications in urban environments [4], or applications that require quadrotor UAVs to transition from indoor-to-outdoor operations or vice versa [5].
The difference between collaborative navigation and cooperative navigation is the ability for the user to independently estimate their accurate position [6]. That is, in cooperative navigation, vehicles help each other to determine their locations, but it is often assumed that each vehicle cannot independently determine its own position to a sufficient level of accuracy [7]. Collaborative positioning techniques, on the other hand, typically use *locally available* or *opportunistic* measurements, such as measurements from neighboring vehicles or civil infrastructure, to help reduce navigation uncertainty. Many collaborative navigation techniques focus on the use of Vehicle Ad-Hoc Networks (VANETs), where multi-sensor fusion is used on individual nodes, and the collaboration between nodes is opportunistic in nature. Most of the collaborative navigation communication is with ranging signal for vehicle-to-vehicle and vehicle-to-infrastructure communication [8] and does not focus on the optimal geometry of the network, but instead use whatever is available. The work herein focuses on cooperative navigation between and Unmanned Aerial Vehicle (UAV) and Unmanned Ground Vehicle (UGV), in which peer-to-peer radio ranging updates are provided by a UGV, and the motion of the UGV is designed to provide the most favorable geometry.

With a similar motivation to the present paper, the use of radiometric signals of opportunity have been shown to improve the estimation navigation and timing information and found that adopting an information-based optimal motion planning performed better than having a pre-described path [9]. The optimal motion planning evaluated different actions that the receiver platform could take, then would move to a location that maximizes the information about the environment. Likewise, the use of a cooperative navigation algorithm to navigate vehicles through a field with obstacles, has been demonstrated in [10]. In this case, a UAV provides a low-resolution map to an UGV, so it can plan its movements based on the objects ahead. Different from previous approaches, in this paper, we do not assume signals of opportunity or cooperation through improved situational awareness, but instead assume active peer-to-peer radio ranging support from a cooperative UGV whose location can be strategically placed.

An additional cooperative navigation system has been developed in Sharma and Taylor [11], where multiple miniature air vehicles (MAVs) work cooperatively to be able to navigate in a GNSS-degraded or denied area. In this case, the MAVs are assumed to be instrumented with IMUs which is shared amongst other MAVs and coupled with range and/or bearing measurement in order to estimate the relative position and velocity of itself and the other MAVs. Similarly, cooperative localization of multiple fixed-wing UAVs in a GPS-denied scenario was also recently
investigated in [12], where dynamic model information, IMU, airspeed, and altimeter data were shared across UAVs to better determine position with updates from a ranging source between each UAV. Similar methods have also been studied on a group of robots [13], in which relative position was estimated by using wheel encoders and multiple cameras to implement bearing-only cooperative localization. In these three works, the use of cooperative navigation was implemented to assist in positioning and the secondary vehicles are moving too. However, the optimization of the secondary vehicles’ position, to improve the positioning of the main vehicle is not implemented as is the focus of the present study.

Several others have considered the use of peer-to-peer ranging to aide a GNSS-degraded navigation solution, such as using stationary nodes [14] [8] or moving nodes [15] [16]. For example, in [14] the best position for placing base stations, such that an area is covered by four base stations at all times, was considered. Likewise, using UWBs on a vehicle-to-vehicle platform, where the vehicles exchanged their position with each other was considered in [15]. Further, the design and evaluation of an estimation strategy for determining the relative pose of the aircraft, between UAVs in a GPS-Challenged [17] or GPS-denied environment [18, 19] has been considered. While these are comparable to the present paper in terms of their exploitation of peer-to-peer ranging, on the contrary, there is not emphasis on the control or design for the location of moving cooperative navigation nodes, which is a focus of this work.

In this paper, a novel cooperative navigation architecture is investigated between a UGV and a UAV, in which a peer-to-peer ranging radio is used to provide range measurements between the two vehicles, and the UGV is strategically moved in order to reduce the navigation estimation error of the UAV. This paper is a significant extension of our prior work in [20], in that we present and investigate a new estimation filter architecture, we incorporate dynamic UAV trajectories, and we improve our modeling of heightened multipath errors. Further, in this paper, two different UGV control strategies are also considered, namely, traversing to a location that is regionally optimal or simply basing the UGV trajectory on the most its most favorable nearby location at each time-step. To offer important insight, a side-by-side comparison of the different estimation architectures and UGV trajectory design approaches is conducted in order to reveal the benefits or downsides of each approach.

The rest of this paper is organized as follows. Section [I] motivates our cooperative approach by assessing the effectiveness of a single ranging source’s ability to reduce PDOP, when given the freedom of the transmitter location. Section [II] describes the algorithm formulations for
the cooperative navigation, including the design of the differential and undifferenced Extended Kalman Filters. Section IV details the two different UGV control strategies investigated. Next, Sections V and VI present the simulation environment used for this work and the results of a series of flight simulations. Finally, Section VII discusses the conclusion of the study.

II. CONCEPT OVERVIEW

Fig. I shows the assumed set-up for the proposed cooperative navigation design within the context of an urban canyon setting. In this case, the UAV is considered to be in a GNSS-challenged environment (e.g., under forest canopy, urban canyon, bridge inspection, etc.). It is further assumed that the cooperative vehicle is not GNSS-challenged. For example, the UGV is assumed to reside in a city intersection with an open-sky access, just outside the forest canopy, not directly under the bridge being inspected, etc.

In this context, (1) a single UGV acting as a ranging source is able to yield a wide range of geometry with respect to the location of a UAV, and (2) a UGV is naturally positioned to improve a UAV’s navigation solution geometry as it emanates its ranging signal from the surface of the Earth, a location that a GNSS transmitter cannot be located. By leveraging these characteristics, this cooperative navigation algorithm is shown to yield significant increases in the accuracy of the positioning of the UAV faced with GNSS-challenged conditions.

A. Review of PDOP

The Dilution of Precision (DOP) metric provides a simple characterization of how user-satellite geometry impacts the positioning errors. In short, it represents how much the random errors of ranging sources scale into the position domain when using trilateration. The more favorable the user-satellite geometry, the lower the DOP metric, and the better the position estimate will be (i.e., assuming that all other error sources are constant). Starting from the Linear Least Squares (LLS) GNSS-solution,

\[
\Delta x = (G^T G)^{-1} G^T \Delta \rho
\]

where \( G \) is the Geometry matrix that is constructed by creating a set of unit-vectors that point along the direction cosines from the user to the satellite locations. The pseudorange measurement model is given by Eq. 2

\[
\rho_C^k = r^k + c\delta t_u + \epsilon_\rho^k
\]
where the vector of the pseudorange measurements is denoted as $\rho$, $r^k$ is the geometric range from the user to satellite $k$, $c$ is the speed of light, $\delta t_u$ is the receiver clock bias, and $\tilde{\epsilon}_\rho^k$ are the measurement residuals. In this LLS solution, it is assumed that the measurement residuals are zero-mean $E[\tilde{\epsilon}_\rho]$, and the variance of the error is given by

$$E[\tilde{\epsilon}_\rho \tilde{\epsilon}_\rho^T] = P_\epsilon = \sigma_{URE}^2 I$$

(3)

$\sigma_{URE}$ is the standard deviation of the User Range Error and is provided by the GNSS control segments [21] and $I$ is the identity. Using the LLS solution for position and clock bias estimation and the zero-mean assumption, the estimation covariance matrix can be formed

$$cov[\Delta x] = \sigma_{URE}^2 (G^T G)^{-1} = \sigma_{URE}^2 H$$

(4)

Using the previous equation, the Horizontal Dilution of Precision, $H^{DOP}$, is formed and shown in Eq. [5]

$$H^{DOP} = (G^T G)^{-1} = \text{diag}[H_{11}, H_{22}, H_{33}, H_{44}]$$

(5)

The Root Mean Square (RMS) of the 3D position is known as the Position Dilution of Precision (PDOP),
Within this study, in addition to GNSS ranging signals, the impact on the Geometry matrix by augmenting the measurement set with a peer-to-peer radio ranging measurement from a cooperative UGV is considered. In this sense, the UGV effectively acts as another satellite observation in which there is an ability to control its location to support favorable geometry.

**B. Reducing PDOP with a Single Ranging Source**

To motivate the potential of this approach, it is important to consider the PDOP reduction that a single additional ranging source could potentially realize. To show this, a simple Monte-Carlo simulation was conducted to determine the maximum amount of PDOP reduction realizable by the addition of a single ranging source across different GNSS satellite geometries. That is, to vary the GNSS satellite constellation geometry, the location of the simulated user’s location and the GNSS time of week were randomized many times for a given scenario. Next, to simulate a GNSS-challenged condition, a high elevation mask was applied to simulate a user’s GNSS visibility being severely impacted. As an example, Fig. 2 shows the percentage of PDOP reduction that could occur with the inclusion of an optimally placed ranging source that is located in the surrounding proximity of the user’s location relative to the GNSS-only PDOP.

In Fig. 2 a 55-degree elevation mask was used to simulate a UAV in the center of a city block. From this analysis, it is apparent that the poorer the satellite geometry, the more benefit the single ranging source can potentially offer. For example, the PDOP can reduced by up to 75% when the GNSS-only PDOP is 6. However, to motivate the need for a cooperative strategy in lieu of simply relying on a UGV placed anywhere, the minimum potential PDOP reduction is also shown, which suggests that a poorly placed UGV could offer no geometric benefit.
Fig. 2: Monte Carlo simulation result with a 55-degree elevation mask, that illustrates the potential improvement of including a single additional ranging source that is optimally placed.

### III. Estimation Algorithm Formulations

This section gives an overview of the estimation filter designs. First, an overview of the undifferenced GNSS EKF design is discussed. Then, the inclusion of a peer-to-peer ranging signal from a UGV is discussed. Next, the differential GNSS EKF design as well as the inclusion of the ranging radio measurement within its measurement update are detailed. Within both filter designs, it is assumed that the UGV is not GNSS-challenged, therefore the absolute position estimate of the UGV is assumed to be known to decimeter-level on-board the UAV via wireless communication. The details of the UGV’s absolute position estimation algorithm are not provided in this paper, and are assumed to have positioning performance accuracy similar to high-quality GPS/INS systems (i.e., $\mathcal{N}(\mu = 0, \sigma = 5\text{cm})$ independently on all 3 axes) [22].

#### A. Undifferenced GNSS EKF Design

Undifferenced GNSS Precise Point Positioning (PPP) is a method that has been used for UAV navigation [23]. The advantages of PPP are it is a global position approach and not needing additional equipment, however, this PPP has a long initialization time [23]. The use
of undifferenced GNSS PPP is investigated here as it would reduce the communication required between the UAV and UGV in comparison to differential approaches.

The PPP approach employs the use of dual-frequency undifferenced GNSS observables. Due to the fact that the PPP approach is undifferenced data, the error sources include the ionospheric delay, tropospheric delay, and receiver clock bias and must be included in the measurement model. To eliminate the ionospheric delay, the ionospheric-free (IF) pseudorange and carrier–phase combinations are employed, which exploits the dispersive nature of the ionosphere to eliminate its impact to first order. The GPS IF combination for pseudorange and carrier–phase are seen in Eq. 7 and Eq. 8 respectively.

\[
\rho_{IF}^k = \rho_{L1}^k \left[ \frac{f_1^2}{f_1^2 - f_2^2} \right] - \rho_{L2}^k \left[ \frac{f_2^2}{f_1^2 - f_2^2} \right] = 2.546 \rho_{L1}^k - 1.546 \rho_{L2}^k \tag{7}
\]

\[
\Phi_{IF}^k = \Phi_{L1}^k \left[ \frac{f_1^2}{f_1^2 - f_2^2} \right] - \Phi_{L2}^k \left[ \frac{f_2^2}{f_1^2 - f_2^2} \right] = 2.546 \Phi_{L1}^k - 1.546 \Phi_{L2}^k \tag{8}
\]

In Eqs. 7 and 8 the \( f_1 \) and \( f_2 \) are the \( L_1 \) and \( L_2 \) frequencies, \( \rho_{L1} \) and \( \rho_{L2} \) are the pseudorange measurements on the \( L_1 \) and \( L_2 \) frequencies, \( \Phi_{L1} \) and \( \Phi_{L2} \) are the carrier-phase measurements on the \( L_1 \) and \( L_2 \) frequencies expressed in units of meters. The superscript \( k \) in Eq. 7 and Eq. 8 is used to denote the measurement between satellite \( k \) and the user. The remaining error sources must be modeled and estimated for each pseudorange and carrier–phase measurement as shown in Eq. 9 and 10 respectively.

\[
\rho_{IF}^k = R^k + c \delta t_u + T_z m(e \ell^k) + \epsilon_\rho^k \tag{9}
\]

\[
\Phi_{IF}^k = R^k + c \delta t_u + T_z m(e \ell^k) + \lambda_{IF} N_{IF}^k + \epsilon_\Phi^k \tag{10}
\]

In Eqs. 9 and 10 \( c \) is the speed of light, \( \delta t_u \) is the receiver clock bias, \( T_z \) is the tropospheric delay in the zenith direction, \( m(e \ell^k) \) is a mapping function dependent on elevation angle, \( \lambda_{IF} \) is the wavelength corresponding to the IF combination, \( N_{IF} \) is phase ambiguity (for the carrier–phase measurement model), \( R^k \) is geometric range between the user and the satellite, as shown in Eq. 11, and \( \epsilon \) are the remaining un-modeled error sources for pseudorange, \( \rho \), and carrier–phase, \( \Phi \), respectively.

\[
R^k = \sqrt{(x^k - x_u)^2 + (y^k - y_u)^2 + (z^k - z_u)^2} \tag{11}
\]
In the above equation, $k$ denotes the satellite and $u$ denotes the user. The troposphere mapping function allows for the reduction of the troposphere delay model to a single unknown delay in the zenith direction with respect to the user, and is scaled according to the user to satellite elevation angles. The troposphere zenith delay, $T_z$, as shown in Eqs. 9 and 10 is composed of both a dry and wet component. The wet delay (i.e., approximately 10% of the total delay) is typically estimated as it is more difficult to model whereas the dry delay is modeled. In this simulation, the Hoppfield model [24] is used to model the wet and dry delay. The mapping function implemented was taken from [25], and is shown in Eq. 12:

$$m(\theta) = \frac{1.001}{\sqrt{0.002001 + \sin(\theta)^2}}$$

(12)

The estimated error-state vector estimated in the undifferenced EKF is shown in Eq. 13:

$$x_{UD} = \begin{bmatrix} \delta r \\ \delta t_u \\ T_w \\ N_{1F}^1 \\ \vdots \\ N_{nF}^n \end{bmatrix}$$

(13)

where the position-error with respect to a nominal guess is denoted as $\delta r$, $\delta t_u$ is the estimated receiver clock bias error with respect to a nominal guess, $T_w$ is the estimated troposphere delay in the zenith direction, and $N_{1F}^1, \ldots, N_{nF}^n$ are the estimated carrier-phase biases of the ionospheric-free carrier-phase data for each satellite in view. For the additional details of the undifferenced GNSS PPP error-state filter formulation adopted, the reader is referred to [22, 26].

B. Undifferenced GNSS EKF Augmented with Peer-to-Peer Radio Ranging

In this study, it is assumed that a peer-to-peer ranging radio devices, such as a pair of Ultra Wide-Band (UWB) radios, are used to realize a Time of Arrival measurement (ToA) [27] between a UGV and a UAV. An advantage of using UWB signals is they have been shown to work in non-LOS application, can penetrate walls, and are not significantly impacted by multipath [15]. Another benefit of using the UWB radios, are their weight, relatively low-power consumption, and low-cost [28].
The standard GNSS observation matrix, $H_{UD}^{obs}$, represents the sensitivity of the measurement models to the state being estimated. The first 3 columns of $H_{UD}^{obs}$ are the partial derivatives of the measurement models with respect to the user’s ECEF position. Column four is the partial derivative with respect to the GNSS receiver clock bias. The troposphere’s zenith delay partials are comprised of the elevation dependent mapping function, and appear in column five of $H_{UD}^{obs}$. The rest of the columns are populated with an identity matrix over the block of rows that correspond to the carrier-phase observations. This identity matrix represents the partial derivative of the carrier-phase observational model with respect to the carrier-phase biases. This is the $H_{UD}^{obs}$ matrix when the UGV is not being used.

The UGV ranging source can be considered similar to an additional GNSS satellite measurement, with the exception that its measurement model only has partial derivatives that are sensitive to the UAV’s position with no sensitivity to the GNSS receiver clock bias, troposphere delay, or GNSS carrier–phase ambiguity. Throughout this paper, the peer-to-peer ranging radio measurement will be defined as, $\upsilon$.

The symbols in the previous equation are denoted as: $n$ is the number of satellites in view, $u^i_X$, $u^i_Y$, and $u^i_Z$ are the unit vector pointing from the user’s nominal position to the $i^{th}$ satellite’s respective X, Y, and Z positions, $\gamma_X$, $\gamma_Y$, $\gamma_Z$ is unit vector pointing from the user’s nominal position to X, Y, and Z location of the UGV. To include the peer-to-peer ranging radio measurement, it must be predicted for inclusion in the filter in order to form an innovation residual. The predicted UAV to UAV peer-to-peer range is formed by the finding distance between the estimated UAV and UGV positions, Eq. 15 and Eq. 16.

$$H_{UD}^{obs} = \begin{bmatrix}
    u^i_X & u^i_Y & u^i_Z & 1 & M^i_{el} & \vdots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & 0_{n \times n} \\
    u^n_X & u^n_Y & u^n_Z & 1 & M^n_{el} & \vdots \\
    u^i_X & u^i_Y & u^i_Z & 1 & M^i_{el} & \vdots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & I_{n \times n} \\
    u^n_X & u^n_Y & u^n_Z & 1 & M^n_{el} & \vdots \\
    \gamma_X & \gamma_Y & \gamma_Z & 0 & 0 & 0_{1 \times n}
\end{bmatrix}$$

(14)

The predicted UAV to UAV peer-to-peer range is formed by the finding distance between the estimated UAV and UGV positions, Eq. 15 and Eq. 16.

$$\dot{\upsilon} = \|\hat{r}_{UAV} - \hat{r}_{UGV}\|_2$$

(15)

$$z_{uwb} = \upsilon - \dot{\upsilon}$$

(16)
In Eq. 15, \( \hat{r}_{UAV} \) is the estimated position position of the UAV, \( \hat{r}_{UGV} \) is the estimated position of the UGV, \( \hat{v} \) is the UWB measured range, and in Eq. 16 \( \bar{v} \) is the UWB predicted range. The overall measurement vector, \( Z_{UD} \), of the undifferenced EKF is given as shown in Eq. 17.

\[
Z_{UD} = \begin{bmatrix}
\Delta \rho_{IF} \\
\Delta \Phi_{IF} \\
\bar{z}_{uwb}
\end{bmatrix}
\]  

(17)

where the \( \Delta s \) denotes the fact the the GNSS measurements are the observed-minus-computed innovation residuals that are formed by differencing the receiver’s observables with their modeled counterparts from Eqs. 7 and 8 which are evaluated about an assumed nominal user position.

C. Differential GNSS EKF Design

The differential GNSS EKF used in this paper incorporates the use of double-differenced satellite observations. Several authors have used Differential GNSS (DGNSS) to increase navigation accuracy. For example, in [29], the authors used DGPS, inertial, and vision sensors for multiple UAV cooperative fault detection. Fewer have used DGPS/UWB in a cooperative navigation setting. For example, within [30] augmented carrier-phase DGPS is coupled with UWB for relative vehicle positioning. In this application, infrastructure points were used to transmit the information of the UWBs, but these points are unable to move. More recently, in [17], the combination of carrier-phase DGNSS with INS and UWB was considered between UAVs, and focused on the need to enhance navigation on the small UAVs that demonstrated fast dynamics and large bank angles which caused degraded GNSS. However, again, none of this prior work focuses on the potential of cooperatively moving one vehicle with respect to another in order to increase navigation observability.

The advantages of using DGNSS over undifferenced PPP are: 1) single-differenced measurements eliminate satellite clock errors, orbit errors, and localized atmospheric errors, and 2) double-differenced measurements eliminate the receiver clock errors. The disadvantage is the need for more equipment, namely a second receiver, to provide the reference measurements to construct double-differences, and a modem to transmit these data. Further, the double-differenced carrier-phase leads to double-differenced carrier-phase ambiguities, \( \nabla \Delta N \), that must be estimated. The estimated state vector for doubled-difference filter is given as shown in Eq. 18.
Double–differenced measurements are constructed by first forming single–differenced measurements of the pseudorange and carrier-phase for each satellite in view, as in Eq. (19), and then differencing each single–differenced measurement with a single-differenced measurement of a chosen reference satellite, as shown in Eq. (20). This process is repeated for both pseudorange and carrier–phase data for both carrier frequencies (i.e., L1 and L2).

\[
\Delta \rho_f = \rho_{UAV,k}^f - \rho_{UGV,k}^f \tag{19}
\]

In Eq. (19) \( \rho_{UAV,k}^f \) is the pseudorange, L1 or L2, of the UAV of each satellite in view, \( \rho_{UGV,k}^f \) is the pseudorange, L1 or L2, of the UGV of each satellite in view.

\[
\nabla \Delta \rho_f = \Delta \rho_f^k - \Delta \rho_f^{refSat} \tag{20}
\]

The single–differenced pseudorange, L1 or L2, is denoted as \( \Delta \rho_f^{refSat,k} \) of the reference satellite.

\[ 
\mathbf{x}_{DD} = \begin{bmatrix} 
\Delta r_{UAV-UGV} \\
\nabla \Delta N_{L1}^{1,refSat} \\
\vdots \\
\nabla \Delta N_{L1}^{N-1,refSat} \\
\nabla \Delta N_{L2}^{1,refSat} \\
\vdots \\
\nabla \Delta N_{L2}^{N-1,refSat} 
\end{bmatrix} \tag{18}
\]

\[ 
\mathbf{Z}_{DD} = \begin{bmatrix} 
\nabla \Delta \rho_{f,L1} \\
\nabla \Delta \rho_{f,L2} \\
\nabla \Delta \phi_{f,L1} \\
\nabla \Delta \phi_{f,L2} \\
\Delta \upsilon 
\end{bmatrix} \tag{21}
\]
where in the case of the differential filter, the carrier-phase measurements are expressed in units of carrier–cycles. To form the observation matrix for the double-differenced EKF, $H_{DD}^{obs}$, there are a few calculations that need to be done:

1) Determine the unit-vector between the reference satellite and the UGV location, as shown in Eq. 22.

2) Determine the unit-vector between each satellite and the UGV, Eq 23.

3) Form the first three columns of $H$ by differencing all satellites (i.e., including the UGV measurement) with the reference satellite’s unit vector, as shown in Eq. 24.

$$
R_{UGV}^{refSat} = \frac{refSat_{xyz} - UGV_{xyz}}{|| (refSat_{xyz} - UGV_{xyz}) ||_2}
$$

$$
R_{UGV}^{Sat} = \frac{Sat_{xyz}^k - UGV_{xyz}}{|| (Sat_{xyz}^k - UGV_{xyz}) ||_2}
$$

$$
R_{refSat}^{Sat,k} = \frac{R_{UGV}^{Sat,k} - R_{UGV}^{refSat}}{|| (R_{UGV}^{Sat,k} - R_{UGV}^{refSat}) ||_2}
$$

As shown in Eq. 25, the rest of the $H_{DD}^{obs}$ matrix is populated with $\lambda_{L1}$ or $\lambda_{L1}$, the GNSS carrier–phase wavelengths, for each row that model a double–differenced L1 and L2 carrier–phase observations. These are need to account for the unknown integer ambiguity parameter estimates.

When the peer-to-peer ranging radio updates are employed, $H_{DD}^{obs}$ only slightly differs. The UGV is considered as an additional GNSS satellite measurement and its position is added to the
end of the observation matrix. \( N \) is the number of satellites observed at that time step.

\[
H_{DD}^{obs} = \begin{bmatrix}
R_{Sat,1,refSat}^X & R_{Sat,1,refSat}^Y & R_{Sat,1,refSat}^Z & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
R_{Sat,n-1,refSat}^X & R_{Sat,n-1,refSat}^Y & R_{Sat,n-1,refSat}^Z & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
R_{Sat,n,refSat}^X & R_{Sat,n,refSat}^Y & R_{Sat,n,refSat}^Z & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
R_{UAV,UGV}^X & R_{UAV,UGV}^Y & R_{UAV,UGV}^Z & \cdots & \cdots & 0_{1 \times (n-1)}
\end{bmatrix}
\tag{25}
\]

\( E. \) Ambiguity Bias Fixing and State Adjustment

Using carrier-phase data requires solving the whole number of cycles, the carrier-phase measurements are from the receiver to each of the visible satellites. The integer part of the state vector is associated with the carrier–phase ambiguities. The parts of the state vector and the error-covariance matrix that are associated with the ambiguity states are input into the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method, created by Peter Teunnisen and the researchers at Delft University, to compute the integer ambiguity set that is most probable [31].

The LAMBDA method uses the Integer Least Squared (ILS) solution for integer estimation based on the float ambiguity vector and the associated variance-covariance matrix. The fixed ambiguities are tested against the ratio of the quadratic forms of residuals of the most-likely and the second most-likely integer candidates. This ratio test, Eq. \( 26 \), is compared to a critical value.

\[
\text{Accept Fix if } \frac{F(N^1)}{F(N^2)} < \frac{1}{C} \tag{26}
\]

The best candidate, \( N^1 \) and the second-best candidate, \( N^2 \), are the two candidates that minimize the quadratic cost function \( F(N) \). \( C \) is the critical value and is set to a constant in this study. In this study, an aggressive ratio was used, 2. The smaller values of \( C \) will lead to more candidates failing, but a less chance of incorrectly accepting a set of incorrect integer ambiguities [31].
IV. COOPERATIVE STRATEGIES

The two strategies that were evaluated include: (1) having the UGV choose the minimum PDOP of the UAV if it were to select from points immediately around the UGV, and (2) having the UGV calculate the minimum PDOP of the UAV if it were to be located anywhere within a grid centered at the UAV, then moving in the direction of this regionally optimal location. These two strategies are dubbed the *Locally Greedy* strategy and the *Regionally Optimal* strategy, respectively. For both approaches, the maximum distance that the UGV is assumed to move over one GNSS measurement updated interval is one meter. Furthermore, for both cooperative approaches, it is assumed that the UAV’s most recent position estimate has been wirelessly communicated to the UGV in order for PDOPs to be evaluated.

A. Locally Greedy Strategy

In this approach, first the UAV receives the signals from all available GNSS satellites to calculate its position solution. After the UAV communicates the satellites it has in-view to the UGV, the UGV determines which location it should move, Eq. [27] in order to reduce the UAV’s PDOP. This is accomplished by the UGV calculating what the UAV’s PDOP would become when incorporating a UWB ranging update from each of the UGV’s candidate locations. With the Locally Greedy approach, the list of candidate positions only includes the positions immediately surrounding the UGV. For this implementation, the number of positions surrounding the UGV were discretized to be the ten points that encircle the UGV’s current location.

To implement this approach, ten candidate UGV heading angles, \( \Psi^{\ell=1:10} = [0, \ldots, 2\pi] \), were selected and candidate positions were calculated using Eq. [27]

\[
\begin{align*}
    r_{k+1}^{UGV, ENU} = \\
    \begin{bmatrix}
        r_{k}^{UGV, E} + d \times \cos(\Psi^{\ell}) \\
        r_{k}^{UGV, N} + d \times \sin(\Psi^{\ell}) \\
        0 \\
    \end{bmatrix}
\end{align*}
\]

where \( r_{k+1}^{UGV, ENU} \) is the UGV’s candidate location for heading angle \( \Psi^{\ell} \), \( r_{k}^{UGV, E} \) is the UGV’s current East position, \( r_{k}^{UGV, N} \) is the UGV’s current North position, \( d \) is the move distance of the UGV, \( \Psi^{\ell} \) is the candidate heading location around the UGV that is being evaluated. With each candidate UGV location, the UAV’s GNSS-only Geometry Matrix, \( G \), is augmented using unit vector to the candidate UGV position and current best estimate of the UAV’s position.

\[
    u_{uwb} = \frac{r_{k}^{UAV} - r_{k+1}^{UGV, ENU}}{\| r_{k}^{UAV} - r_{k+1}^{UGV, ENU} \|_2}
\]
where the $u_{uw}$ is the unit vector distance between the UAV and the candidate UGV position. With the set of UAV Geometry matrices augmented with each UGV candidate location, the PDOP for each candidate 1 to $\ell$ is evaluated, and the minimum PDOP is selected as indicated in Eq. 29

\[
PDOP_{min} = \text{argmin}(PDOP_1, \ldots, PDOP_N)
\]

Once the minimum PDOP of the UAV is identified, the UGV is moved to the location that corresponds to the minimum UAV PDOP.

Additional UGV path planning logic was also included to ensure that it does not get too close to the UAV. This is included to ensure that the UGV doesn’t also enter the GNSS-challenged environment, itself. For the time being, this is implemented as a simple 70 meter square around the UAV’s best known location as a ‘no-UGV-zone’. As such, if the UGV’s next desired trajectory position falls inside the perimeter, the following steps are taken. First, the slope of the distance between the UAV and UGV is found using Eq. 30.

\[
m = \frac{\Delta r_N}{\Delta r_E}
\]

where $m$ is the slope, $\Delta r_N$ is the North component of the distance between the UAV and UGV, and $\Delta r_E$ is the East component of the distance between the UAV and UGV. Next, the intersection of the perimeter and the UGV, is determined, based on the slope and the equation for a circle as shown in Eq. 31 and 32.

\[
r_{UGV,E}^{k+1} = \text{sign}(r_{UGV,E}^k)\sqrt{r_{perim}^2 \left( m^2 + 1 \right)}
\]

\[
r_{UGV,N}^{k+1} = r_{UGV,N}^k
\]

where $r_{UGV,E}^{k+1}$ is the UGV’s next East position, $r_{perim}$ is the radius of the perimeter, and $r_{UGV,N}^{k+1}$ is the UGV’s next North position. In Eq. 31, the sign operator is to ensure UGV is located in the proper quadrant of the circle. For future development of this approach, a priori map information will be included in this part of the trajectory design for selection of the perimeter.

**B. Regionally Optimal Strategy**

For the Regionally Optimal cooperative UGV path planning strategy, a square grid is setup with the UAV at the center. Then, peer-to-peer-range-augmented-PDOP from including a ranging observation emanating from every point on the grid is computed. As an example of this approach,
Fig. 3 shows the percentage reduction possible for the square grid at one time step, for one simulation scenario. The yellow represents a region in which a 70% percentage reduction is achievable. After evaluating this grid, the minimum overall PDOP is determined as seen in Eq. 33.

\[ PDOP_{min} = \arg\min_{1 \leq i \leq N_{grid}}(PDOP_{i_{grid}}, \ldots, PDOP_{N_{grid}}) \] (33)

Once the east and north location of where the PDOP is minimum is found, the UGV is driven in that direction. This is accomplished by first determining the distance between the current UGV position and the location where the PDOP of the UAV is minimum over the grid as shown in Eq. 34 and 35.

\[ \Delta r^{UGV,E} = r^{UGV,E}_k - E_{PDOP_{min}} \] (34)

\[ \Delta r^{UGV,N} = r^{UGV,N}_k - N_{PDOP_{min}} \] (35)

where \( E, N_{PDOP_{min}} \) is the location where the PDOP would be minimized for the UGV location within the grid. From here, the heading angle, \( \Psi \), is found to determine which direction the UGV should move, as seen in Fig. 4. The heading was calculated by Eq. 36.
Previous UGV Location

Current UGV Location

\[ \Psi = \arctan \left( \frac{\Delta r_{UGV,N}}{\Delta r_{UGV,E}} \right) \] (36)

Since there is a constraint that the UGV can only move a maximum of one meter per time step, the next UGV location is determined based on the move distance and the heading, Eq. 37 and 38.

\[ r_{UGV,E}^{k+1} = r_{UGV,E}^k + d \times \cos(\Psi) \] (37)

\[ r_{UGV,N}^{k+1} = r_{UGV,N}^k + d \times \sin(\Psi) \] (38)

where \( d \) is the maximum move distance of the UGV and \( \Psi \) is the heading angles. As stated above, there is a check in place to make sure the UGV does not come too close to the UAV. If it does, the same procedure described in Section IV-A.

V. Simulation Environment

The raw GNSS data used to simulate the data for this study was generated using a commercially available SatNav–3.04 Toolbox [32], which is a GNSS constellation simulation toolbox.
In each simulated flight, the UAV was simulated to be moving at a constant 0.5 \( m/s \) and the UGV is permitted to move a maximum distance of 1 meter each optimization update (i.e., 10 Hz updates). Simulation configuration inputs were defined for the generation of GPS signals and error sources, and several parameters were randomized for each simulation scenario including the simulation location (i.e., latitude, longitude, and height), the time of the GPS week, and the length of the flight. These inputs were selected at random in order to give each flight a different GNSS satellite constellation geometry and atmospheric effects. For more information on the generation of data, please refer to [22] for a more detailed description. The scale of the GPS error sources, used in the simulation, are listed in Table I.

<table>
<thead>
<tr>
<th>Error-Sources</th>
<th>Model Parameters</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric Delay</td>
<td>First order ionospheric effects mitigated with dual-frequency</td>
<td>linear scale factor randomly selected between [0.7,1]</td>
</tr>
<tr>
<td>Multipath</td>
<td>1.0 intensity: ( \sigma = 8m, \tau = 2min )</td>
<td>linear scale factor randomly selected between [0,2]</td>
</tr>
<tr>
<td>Orbits</td>
<td>Orbits ( \sigma = 5cm )</td>
<td>Description provided in paragraph below</td>
</tr>
<tr>
<td>Phase Ambiguity</td>
<td>Random initialization and phase breaks correlated with UAV attitude</td>
<td>likelihood varied from [0.008,0.02]</td>
</tr>
<tr>
<td>Receiver Clock Bias</td>
<td>Initial Bias ( \sigma = 30ns, \delta \tau_b = 100ns )</td>
<td>N/A</td>
</tr>
<tr>
<td>Thermal Noise</td>
<td>( \sigma_\rho = 0.32m, \sigma_\phi = 0.16\lambda )</td>
<td>linear scale factor randomly selected between [0,1]</td>
</tr>
<tr>
<td>Tropospheric Delay</td>
<td>Percent of error assumed handled by broadcast correction</td>
<td>Modified Hopfield with linear scale factor randomly selected between [0,1.5]</td>
</tr>
</tbody>
</table>

A GNSS error-source particularly important to this study was GNSS multipath errors. As such, for this work the multipath error was increased to simulate the GNSS-challenged environment of an urban-canyon. Multipath was modeled as a first order Gauss-Markov error source and with a \( \sigma = 8 \) meters and a time constant, \( \tau \) of 2 minutes. To simulate an urban canyon, multipath was set as a function of the elevation of the satellite and added to the original multipath formed in the simulation, seen in Eq. [39].
where $m_{pUC}$ is the multipath in an urban canyon, $m_p$ is the multipath modeled, and $el^k$ is the elevation of the satellite.

Furthermore, to simulate a GNSS-challenged environment, a high elevation mask, i.e. buildings in an urban canyon, was incorporated as seen in Fig. 5. The masks were held constant throughout the simulated flight, and for all data sets. The top figure shows the elevation and azimuth of the satellites seen in one specific simulation. The blue line shows the progression of the azimuth and elevation of the satellites without a mask and the red line shows the satellites’ elevation and azimuth after the high elevation mask was imposed. The bottom plot shows what the PDOP was before and after the mask, with the blue representing the PDOP before a mask was applied and the red line representing the PDOP after a mask was applied.

For the peer-to-peer ranging radio measurements a zero-mean white-noise error of 5 cm was added to the true range, as commercially available UWB systems can achieve 2 cm ranging [33]. This simulation also included an orbit error model to represent the errors in the GNSS broadcast ephemeris. The satellite ephemeris errors were modeled by differencing the broadcast products...
provided by the International GNSS Service (IGS) and the Center for Orbit Determination (CODE). A multi-sinusoidal model was fitted to the error, and based on the time of day. More information can be found in [22]. The UAV and UGV were assumed to start at arbitrary positions. Within this study, following this Monte-Carlo design, 25 flight data sets were generated.

VI. RESULTS

Each of the 25 generated flight data sets were run through estimation filter’s both with and without the aiding from the UGV in order to characterize the performance of including the cooperative strategy. In addition, both the undifferenced EKF and a double differenced EKF were employed on each data set. Likewise, both of the cooperative UGV trajectory design strategies were tested for each data set in order to uncover any differences between the two approaches.

The first few figures detail a specific example of one simulation trial in which the UGV’s path and the reduction in PDOP are shown. For example, Fig. 6 shows the path that the UGV takes when employing both the Locally Greedy path (left panel) and the Regionally Optimal (right panel) path planning algorithm.

The green $\ast$ is where the UGV starts, the black $O$ is where the Locally Greedy path ends, and the yellow $x$ is where the Regionally Optimal path ends. The orange line indicates the Locally Greedy path, and the black line indicates the Regionally Optimal path. The blue $\ast$ indicates where the UAV starts and the red $X$ is the end of the UAV path. The blue line shows the entire path of the UAV. The Locally Greedy UGV moves toward the optimal location in the East direction and moves back and forth in the east direction, while also moving in the north direction. Whereas the Regional Optimal strategy moves toward the optimal geometric location, and then moves along the constraint boundary. This is expected, as we can see from Fig. 3, the largest reduction of PDOP matches the path of the UGV with Regionally Optimal strategy employed for this particular scenario.

Fig. 7 shows the PDOP over the entire flight for having no cooperative UGV, the UGV employing the Locally Greedy path planning algorithm, and the UGV employing the Regionally Optimal approach. The PDOP for the Regionally Optimal algorithm is reduced further than that of the Locally Greedy path planning approach, and both are significantly lower than the case of including no cooperative UGV. As shown in Fig. 8, this extra cooperative ranging measurement and PDOP reduction leads to more accurate, smoother, and more consistent positioning performance.
Fig. 6: Flight simulation example of the UAV with the cooperative UGV using a Locally Greedy (Left Panel) and Regionally Optimal (Right Panel) path planning strategy. The no-UGV boundary at the time of the UAV start is shown as a black rectangle,

To summarize the positioning performance result for the undifferenced GNSS EKF, Fig. 9 shows the median 3D position error for each strategy over the 25 flight simulation trials. In general, the solution improves when using cooperative ranging from a UGV, and the Regionally Optimal approach provides the best solution with a nearly 40% error reduction.

Next, in Fig. 10 summarizes the median errors of differential GNSS EKF, with and without cooperative UGV aiding are shown, where both integer ambiguity fixed and float solutions are shown.

As expected, in all three cases, the carrier–phase ambiguity integer fixed solution performs better when compared to the carrier–phase ambiguity integer float solution. Also, having a UGV significantly improves the solution with the Regionally Optimal UGV path planning strategy proving to be the best approach. Table II summarizes both the median and average 3D position error for each strategy over the 25 flights. These metrics are shown alongside the PDOP and % of Carrier-Phase Biases successfully integer fixed.

In Table II it is clear having a cooperative UGV employing the Locally Greedy strategy is
Fig. 7: Example flight trial of the UAV’s PDOP without UGV, with a Locally Greedy UGV, and with a Regionally Optimal UGV.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Strategy</th>
<th>Med. Error (m)</th>
<th>Avg. 3D Error (m)</th>
<th>Avg. PDOP</th>
<th>Avg. % Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undiff.</td>
<td>None</td>
<td>1.17</td>
<td>2.49</td>
<td>4.66</td>
<td>N/A</td>
</tr>
<tr>
<td>Undiff.</td>
<td>Greedy</td>
<td>0.77</td>
<td>0.79</td>
<td>2.33</td>
<td>N/A</td>
</tr>
<tr>
<td>Undiff.</td>
<td>Optimal</td>
<td>0.70</td>
<td>0.71</td>
<td>1.58</td>
<td>N/A</td>
</tr>
<tr>
<td>Diff. Float</td>
<td>None</td>
<td>0.64</td>
<td>1.86</td>
<td>4.66</td>
<td>N/A</td>
</tr>
<tr>
<td>Diff. Fixed</td>
<td>None</td>
<td>0.23</td>
<td>1.55</td>
<td>4.66</td>
<td>99.94</td>
</tr>
<tr>
<td>Diff. Float</td>
<td>Greedy</td>
<td>0.30</td>
<td>0.42</td>
<td>2.35</td>
<td>N/A</td>
</tr>
<tr>
<td>Diff. Fixed</td>
<td>Greedy</td>
<td>0.19</td>
<td>0.30</td>
<td>2.35</td>
<td>99.98</td>
</tr>
<tr>
<td>Diff. Float</td>
<td>Optimal</td>
<td>0.20</td>
<td>0.28</td>
<td>1.56</td>
<td>N/A</td>
</tr>
<tr>
<td>Diff. Fixed</td>
<td>Optimal</td>
<td>0.10</td>
<td>0.14</td>
<td>1.56</td>
<td>99.99</td>
</tr>
</tbody>
</table>

TABLE II: Summary of positioning performance with and without UGV cooperative navigation, for both control strategies and both filter designs.
better than having no UGV, and that having a UGV that employs the Regionally Optimal strategy is the best scenario in terms of positioning performance. This is seen in both the undifferenced GNSS EKF and the differential GNSS EKF. Of particular significance, in Table II it is evident that the 3D position error can be reduced from a median error of 1.17 m to 10-cm when using a cooperative UGV. Additionally, when comparing the average to median performance over the 25 flights, the use of the cooperative UGV makes the positioning performance much more consistent despite the GNSS-challenged conditions. That is, with the UGV, the average and medians 3D errors are very similar, where as there is a large discrepancy between the average and median performance without a UGV. Further, the benefit of the cooperative UGV is clear in the average PDOP, which is significantly reduced by cooperative UGV. In particular, the average PDOP reduction over the 25 flights was 50% with the Locally Greedy strategy and a PDOP reduction of nearly 2/3 with respect to the GNSS-only value whenever the UGV is steered to the Regionally Optimal location. Finally, while a marginal impact, it also is promising that whenever
Fig. 9: 3D Position Error of Median of RSOS data sets with the Undifferenced EKF

Fig. 10: 3D Position Error of Median of RSOS data sets with the Double Differenced EKF
the cooperative UGV is present, the percentage of epochs that are successfully integer ambiguity fixed is greater with a UGV when compared to not using a UGV. It should be noted that this metric would be more favorable for the cooperative UGV if a more stringent ambiguity ratio test was used.

In addition to presenting the statistical summary of the 25 flights, Fig. 11 shows the 3D positioning performance for each of the 25 flights for both filters and all three scenarios, no UGV, Locally Greedy, and Regionally Optimal, as well as all three filter types (i.e., undifferenced, differential float, and differential fixed). As shown in Fig. 11, having a UGV consistently helps in reducing the 3D positioning error of the UAV for the majority of flight trials. In nearly all cases, the addition of the cooperative UGV makes a significant impact, while there are a few cases that its impact is negligible. In the flights that the UGV benefit was negligibly, the GNSS-only geometry was likely favorable as is, as is evident in that the fact that positioning error is already low without the cooperative UGV in these instances. Likewise, as suggested in the motivational Monte-Carlo study of the potential PDOP reduction from a single ranging source that is shown in Fig. 2, the worse the GNSS-only performance, the more beneficial the updates
from the cooperative UGV. This is evident in particular, in flight trials # 19 and #24.

It is worth also noting that there are practical advantages and disadvantages of each approach on the implementation–level. For example, undifferenced GNSS EKF requires less communication and the computational performance is less than the differential GNSS EKF (i.e., when including the LAMBDA method, etc.). However, the differential GNSS EKF has been shown to be more precise in all cases (i.e., Locally Greedy UGV, Regionally Optimal UGV, and No UGV). Likewise, with respect to the two cooperative planning strategies of the UGV, the Regionally Optimal performed better in both filters, but the computational time to compute a grid around the rover is much greater than compared to the Locally Greedy strategy. As such, depending on the required accuracy and computational constraints of the application, one configuration may be a better fit than others.

VII. CONCLUSIONS

A novel cooperative navigation strategy has been developed and evaluated in simulation and has been shown to dramatically improve the positioning performance of a UAV in a GNSS-challenged environment. This paper has described two different filter designs, two different cooperative planning techniques, and the results of several flight simulation trials. It has been shown that having a single UGV ranging–source that is strategically located can significantly help in the positioning of the UAV in unfavorable GNSS conditions. Further, it has been shown that the more challenged the GNSS environment is, the more benefit a cooperative UGV offers. An important conclusion of this study is that when comparing strategies, it has been shown that using differential GNSS estimator formulation coupled with the Regionally Optimal UGV strategy is the best. Using this configuration, the 3D positioning error can be reduced from meter-level (1.17 meters 3D error median) to cm-level (10 cm 3D error median), which is the required level of precision for several important applications. However, it is important to note that, even when using the most simple cooperative configuration (i.e., undifferenced filter and locally greedy cooperative strategy) the average PDOP was still reduced by 50% leading to an average position reduction from an average 3D error 2.5 meters to less than 0.8 meters. When optimally placed, the average PDOP is further reduced, for an overall reduction of about 77% on average. Future work will consider an experimental flight-test evaluation of the proposed cooperative navigation strategies, and investigating the use of the UAV’s entire planned mission trajectory for UGV path planning.
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